

Kuwait University	Math. 101 (Incomplete)	November 10, 1997
Dept. of Math. & Comp. Sci.	First Mid-Term Exam	Duration: 75 minutes

Answer all of the following questions. Each question counts 5 points.

Calculators are not allowed

1. Evaluate the following limit (if it exists)  $\lim_{x \rightarrow 0} \frac{\cos(2x) - \cos(4x)}{x^2}$ .

2. Let  $f$  and  $g$  be two functions defined on  $(-\infty, +\infty)$ . If  $12 \leq g(x) + f(x) \leq 3x^{\frac{2}{3}}$ , for all  $x$ , and  $\lim_{x \rightarrow 8} g(x) = 2$ , find  $\lim_{x \rightarrow 8} \sqrt[3]{x} f(x)$ .

3. Find the horizontal and the vertical asymptotes (if any) for the graph of the function  $f(x) = \frac{5x^2 + x}{|x|(x-2)}$ .

4. Let  $f$  be given by :

$$f(x) = \begin{cases} 3x^2 + 3x + 1 & , \text{if } x < -1 \\ -x^3 & , \text{if } -1 \leq x \leq 1, \\ x^3 - 6x^2 + 6x - 2 & , \text{if } x > 1. \end{cases}$$

Discuss the differentiability of  $f$  at  $x = -1$  and at  $x = 1$ .

5. Show that the equation

$$x^3 \cos(\pi x) + 3 \sin^2\left(\frac{\pi x}{2}\right) - 1 = 0$$

has a real solution.

6. If  $f(x) = \sin(5x)$ . Use the definition of the derivative to find  $f'(\pi)$ .

7. Let  $f(x) = (x-1)^{\frac{2}{5}}(x-2)^{\frac{1}{3}}$ . Find all points at which the graph of  $f$  has a vertical tangent line or a cusp.

8. Find the derivative of the function  $f(x) = \sqrt[3]{x^2} \cos(\pi - x^2) + \sec^2(x^3)$ .

Good Luck